Anamorphic Stereoscopic Imaging

Exploiting nonlinear projections for wide-angle depth generation

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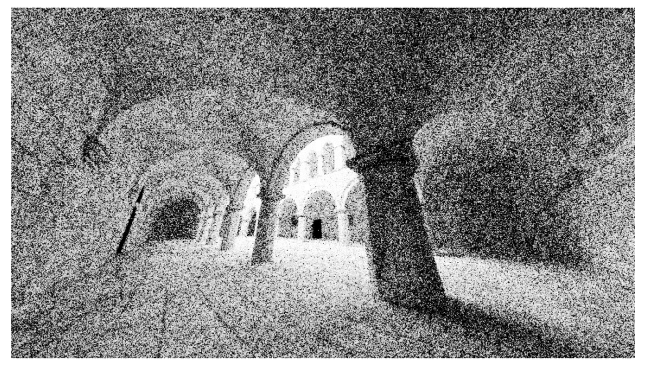
Motivation

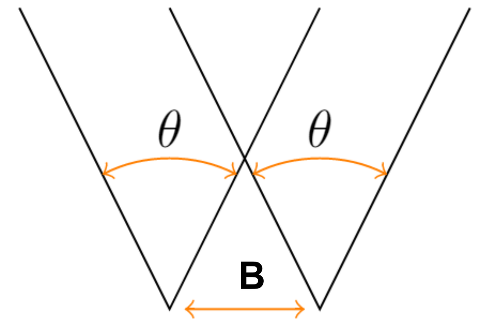
Traditional lenses used for computational depth generation are known as perspective projection, or rectilinear lenses. Their projected images preserve straight lines, and preserve constant space sampling across the image. However, they present at least three main challenges.

- **Natural vignetting –** due to the geometry of perspective projection lenses [AHA01], they will tend to exhibit light falloff as a function of cos(θ) to the fourth power. This makes wide angle lenses impractical, as even at 60 degrees off-axis, a rectilinear lens gather only 6.25% the same light intensity as on axis.
- **Infinite Sampling** – due to the requirement to have constant sampling in real-world space, rectilinear lenses require tan(θ) pixels to capture areas θ degrees off axis. This explodes towards infinity as θ approaches 90 degrees. But even at smaller angles, rectilinear lenses begin to devote a dispropriate amount of their pixel area to the edges of their frame.
- **Stereo Accuracy** -- While the first two points are applicable to almost all problems in imaging, there 's a special case of interest for this work – stereoscopic depth imaging. Traditional stereoscopic imaging performs significantly worse at longer distances than close ones. This is due to the geometry of the lens, and by changing to a different projection model, it may be possible to change these accuracy properties.


A rectilinear lens projection

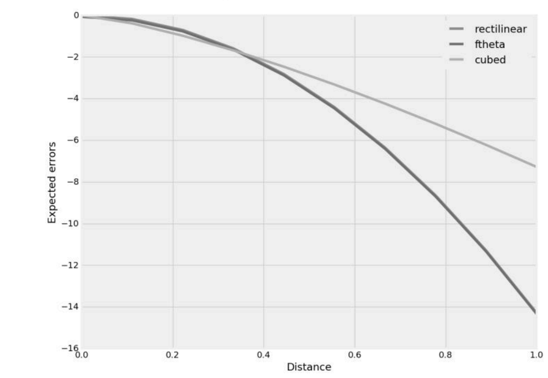

An f- θ lens projection

New Technique



Two dimensional overview of our coordinate system. The field-of-view of each camera is identical and equal to θ. The spacing between the cameras is B.

Error with respect to distance for different lens projections at u = 1% of focal length



Derivation of stereoscopic system constraints

To parametrize the equations of accuracy and correspondence, we can setup a general equation to define stereosopic correspondence between two imagers as follows

$p(x) = p(x-d) + B.$

Where p(x) is the projection function from the image, and d is the disparity

A traditional rectilinear lens model would be

$p(x) = f \cdot \tan(\theta) = f \ * \ \frac{x}{z}$

Combining the two gives us

$z = \frac{f \cdot B}{d}$

Additionally, we can take the derivative, $\frac{\partial z}{\partial d}$ and substitute into our expression to generate an error relationship. Traditionally we take the error relative to a single pixel disparity error ($\partial d = 1$).

$\frac{\partial z}{\partial d} = -\frac{z^2}{f \cdot B}\partial d$

And it becomes clear that rectilinear lenses will suffer from quadratic depth error given constant disparity space errors.

Stereoscopic constraints for other lenses

Repeating the previous derivations for additional lens geometries gives us the following table

Equation	World-to-Image	Image-to-World	Depth	Depth Error
Rectilinear	$u = f \cdot tan(\theta)$	$z\frac{u}{f}$	$\frac{Bf}{d}$	$-\frac{z^2}{Bf}$
$f\theta$	$u = f \cdot \theta$	$z\tan\left(\frac{u}{f}\right)$	$\frac{B}{\tan\left(\frac{u}{f}\right)+\tan\left(\frac{1}{f}(d-u)\right)}$	$-\frac{1}{Bf}\left(z^2+\left(B-z\tan\left(\frac{u}{f}\right)\right)^2\right)$
Cubic	$u = (f \cdot tan(\theta))^{\frac{1}{3}}$	$z\frac{u^3}{f^3}$	$\frac{Bf^3}{u^3+(d-u)^3}$	$-\frac{3z^2}{Bf^3}\left(\frac{Bf^3}{z}-u^3\right)^{\frac{2}{3}}$
Orthographic	$u = f \cdot sin(\theta)$	$\sqrt{\frac{u^2z^2}{(f-u)(f+u)}}$	Unable to derive	Unable to derive

Of particular interest is the cubic depth error, which can be rewritten as

$$\frac{\partial z}{\partial d} \propto \begin{cases} z^{\frac{4}{3}}, & if \ \frac{Bf}{z} \gg u^3 \\ z^2 \cdot u^2 & if \ u^3 \gg \frac{Bf}{z} \end{cases}$$

That is, for the center of the image, cubic lenses will tend to have sub-quadraic depth error. For the rest of the image, they will tend to have traditional stereo error that gets quadratically worse with as one samples off-axis.

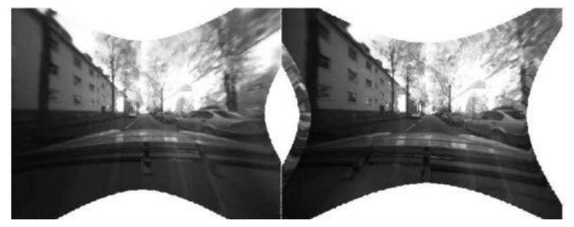
Related Work

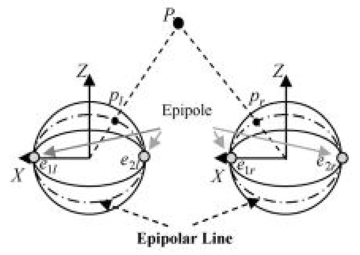
There have been many existing methods on expanding stereoscopic correspondence algorithms to handle non ideal lenses. Unfortunately they tend to fall in two buckets.

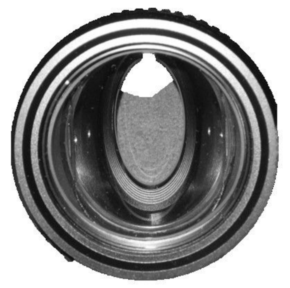
First, many authors [AF05]. [GNT98] [Geh05] use wide angle lenses to perform stereo matching, but they distort the image back to a rectilinear projection, forcing large scale resampling of the image and abandoning the beneficial properties of using a nonlinear lens, such as the ability to image an entire hemisphere.

Lastly, Li [Li06] [Li08] developed methods of using nonlinear lens for generating depth, including calibration and search, but the method only applied to only spherical projections. Additionally, their use of nonlinearities was limited to only the axis perpendicular to the epipolar line.

Popular in the cinematic domain for mapping wide aspect ratio images onto a square segment of film, anamoprhic lenses distort only one axis of the image. For stereoscopic imaging, projection along the axis perpendicular to the epipoles has no impact on stereo depth error





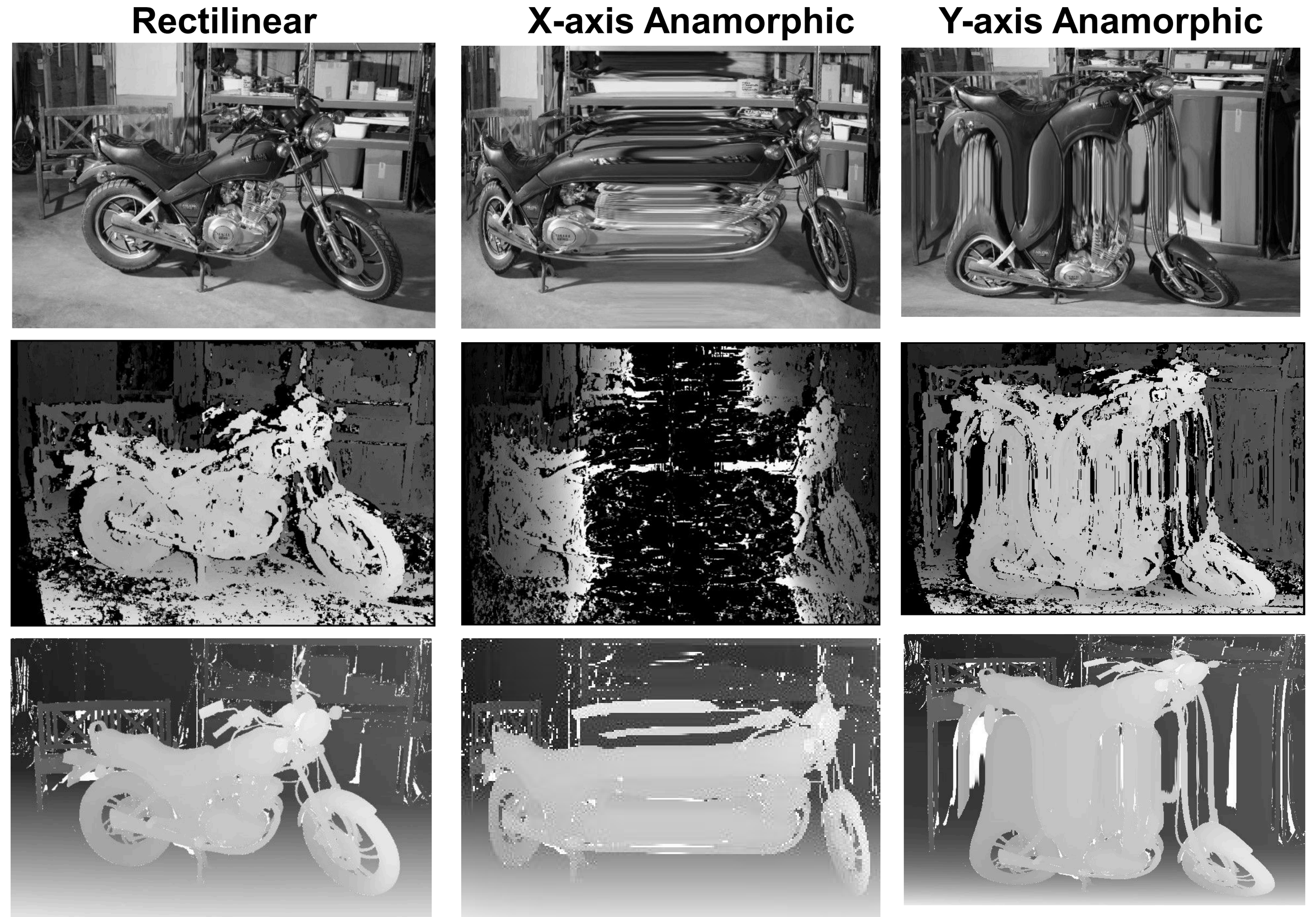


[Fle95] Margaret M Fleck. "Perspective projection: the wrong imaging model". In: Research report Kingslake 1992 (1995), pp. 95–01.
[GNT98] J M Gluckman, S K Nayar, and K J Thoresz. "Real-Time Omnidirectional and Panoramic Stereo". In: DARPA Image Understanding Workshop (IUW).
[AHA01] Manoj Aggarwal, Hong Hua, and Narendra Ahuja. "On cosine-fourth and vignetting effects in real lenses". In: Computer Vision, 2001. ICCV 2001.
[Sha02] Shishir Shah. "Depth Estimation Using Stereo Fish-Eye Lenses". In: Image Processing, 1994. Proceedings. Vol. 2. IEEE. 2002, pp. 740–744. ISBN: 0818669500.
[AF05] Steffen Abraham and Wolfgang Forstner. "Fish-eye-stereo calibration and ¨epipolar rectification". In: ISPRS J Photogramm 59.5 (2005),
[Geh05] Sk Gehrig. "Large-field-of-view stereo for automotive applications". In: Omnivis 2005 (2005)
[Li06] Shigang Li. "Real-Time Spherical Stereo". In: 18th International Conference on Pattern Recognition (ICPR'06). Oct. 2006,
[Li08] Shigang Li. "Binocular Spherical Stereo". In: TITS 9.4 (Dec. 2008), pp. 589–600. ISSN: 1524-9050. DOI: 10.1109/TITS.2008.2006736.
[Kit11] Nobuyuki Kita. "Dense 3D Measurement of the Near Surroundings byFisheye Stereo". In: 2011, pp. 148–151.

Experimental Results

Results using a standard stereo dataset

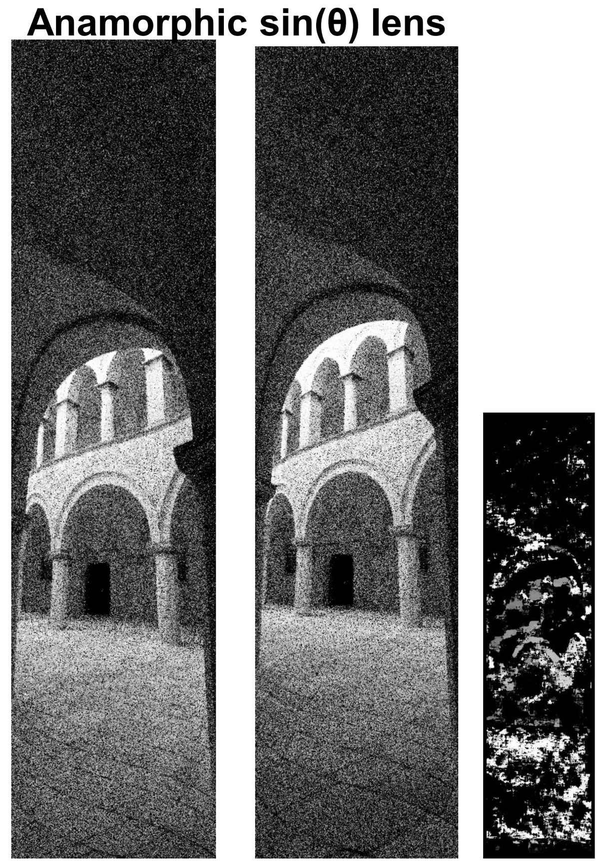
Since a cubic lens requires very dense pixel sampling (1% off axis pixel resolution would require 3x^2 sampling, or 30x super-sampling), we were having difficulty finding sufficiently dense textured scenes, so we resampling a high-resolution stereo dataset



Results using a rendered scene

To demonstrate the value of using an anamorphic, non-linear lens, we generated synthetic images with realistic vignetting and the appropriate lens projections. We then ran a simple stereo matching algorithm on both images. Unfortunately, the low resolution and noisy path-traced input generates noisy depth maps for both test cases. Experiments are on-going to use this as a basis for quantitative comparison.

Anamorphic sin(θ) lens



Rectilinear lens